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Statistics in the Middle Grades:

Understanding Center and Spread

Youngsters and adults alike are confronted daily with situations involving statistical information. Making sense of data and dealing with uncertainty are skills essential to being a wise consumer, an enlightened citizen, and an effective worker or leader in our data-driven society. The importance of statistics education as an integral part of the mathematics curriculum was signaled by NCTM in its *Curriculum and Evaluation Standards for School Mathematics* (1989). Subsequent recommendations in state frameworks, NCTM's *Principles and Standards for School Mathematics* (2000), and most recently in the *College Board Standards for College Success: Mathematics and Statistics* reflect consistent attention to data analysis, probability, and statistics across the grades.

In 2007, the American Statistical Association released a report titled *Guidelines for Assessment and Instruction in Statistics Education* (GAISE). That report provides learning trajectories for key ideas of statistics organized into three developmental levels, A, B, and C. Although these three levels may parallel the standard grade-level bands (elementary, middle, and secondary), they are based on students' prior statistical experiences rather than on grade level. The three companion articles in this month's issues of *Teaching Children Mathematics*, *Mathematics Teaching in the Middle School*, and the *Mathematics Teacher* illustrate how the GAISE report can be used to shape a coherent development of basic ideas related to the distribution of a data-based variable beginning with exploring distributions of data, then developing an understanding of center and spread, and finally building sound reasoning under uncertain conditions. For a perspective on K–12 statistics education, read all three articles in the series.

This department provides a forum to stimulate discussion on contemporary curricular issues across a K–12 audience. NCTM plans to publish sets of three articles, focused on a single curriculum issue. Each article will address the issue from the perspective of the audience of the journal in which it appears. Collectively, the articles are intended to increase communication and dialogue on issues of common interest related to curriculum. Manuscripts on any contemporary curriculum issues are welcome. Submissions can be for one article for one particular journal, or they can be for a series of three articles, one for each journal. Submit manuscripts at the appropriate Web site: tcm.msubmit.net, mtms.msubmit.net, or mt.msubmit.net, or contact editors Barbara Reys (reysb@missouri.edu) for *TCM*, Glenda Lappan (glappan@math.msu.edu) for *MTMS*, or Chris Hirsch (christian.hirsch@wmich.edu) for *MT*.

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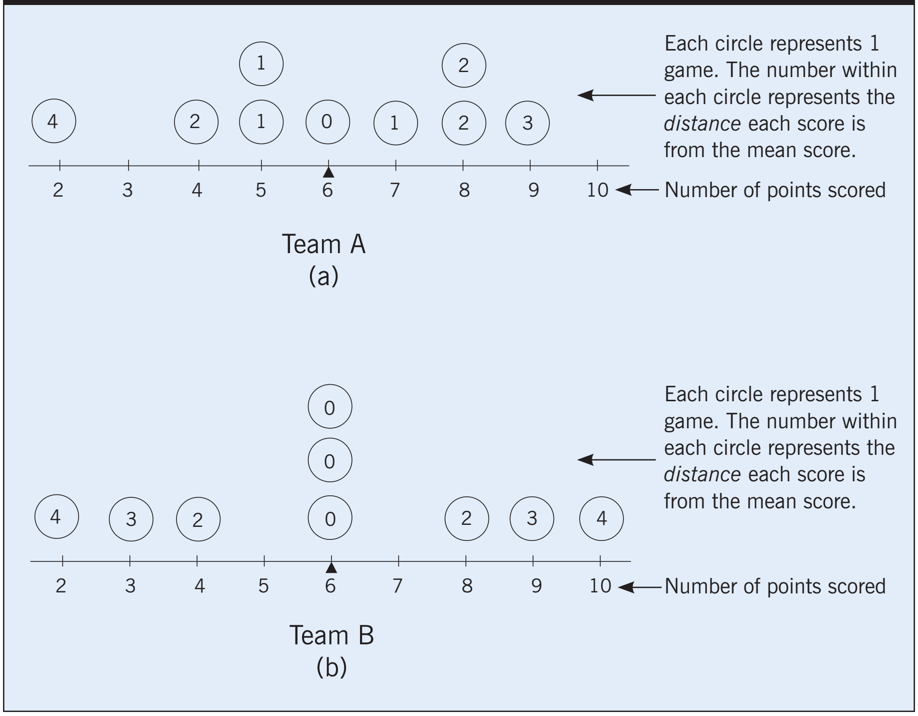
The GAISE report emphasizes the importance of students having experience with statistical thinking throughout the pre-K–12 curriculum. Students' encounters with statistics in the middle grades should build on their foundational experiences from the elementary grades and provide a link to the inferential types of statistical thinking developed at the high school level. Middle-grades students should be actively involved in the statistical problem-solving process described in the GAISE report. That process involves (1) formulating a question that can be addressed with data, (2) collecting data to address the question, (3) analyzing the data, and (4) interpreting the results.

As students transition between levels A and B, they begin to rely less on data representations that display individual outcomes (e.g., line plots, dot plots, picture graphs) and to use representations based on groupings of the data and numerical summaries of data (e.g., histograms and box plots). From these representations, new questions arise: "Where is the center of the data located?" "How spread out are the data?" "Where do the data cluster?" Many representations for numerical data developed at level B focus on central location and the spread (degree of variability) in the data.

Fig. 1 Total points scored in each of nine games



THE MEAN AS THE BALANCE POINT: VARIATION IN DATA FROM THE MEAN

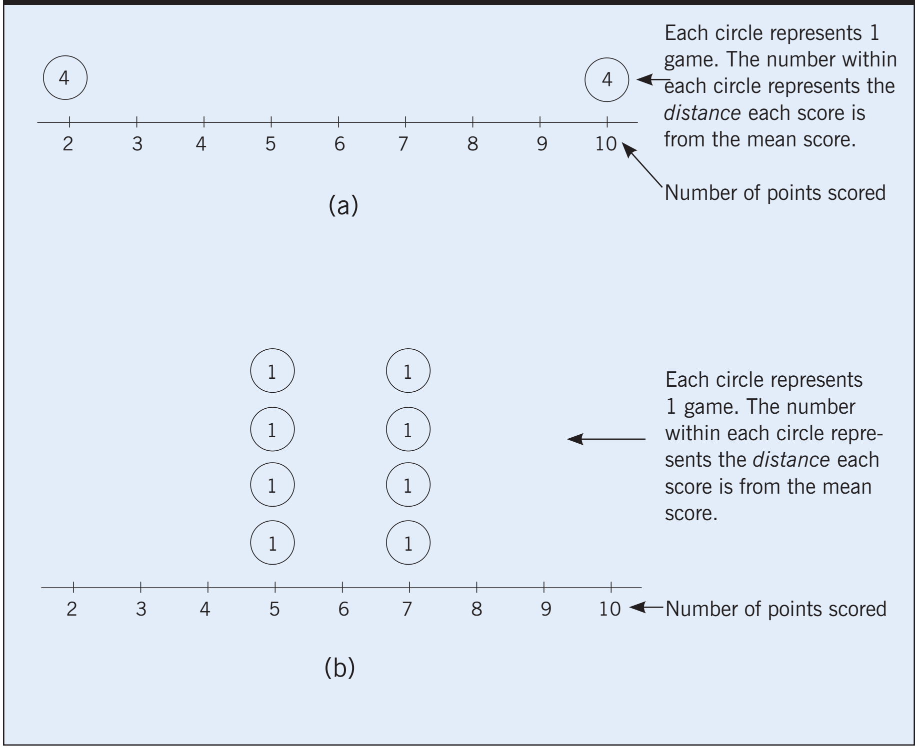
In the companion article to this one in *Teaching Children Mathematics,* Franklin and Mewborn (2008) describe the mean as the "fair share" value for a collection of discrete numeric data and the "number of steps to fair share" as a measure of variation in the data that are developed for level A. This article presents level B examples that build on the ideas in the *TCM* article, expands the notion of the mean to include the concept of a balance point for the data distribution, and develops an alternative measure of variation about the mean. (A similar activity is described in the GAISE report [Franklin et al. 2007].)

The dot plots (line plots) in **figures 1a** and **1b** show data from the Franklin article for two sets of scores for nine soccer games. Each set of scores has a mean value of 6. The number within each circle represents the *distance* each score is from the mean score. Notice in each plot that the total distance from the mean is the same for the values above the mean as for the values below the mean. For **figure 1a,** the total distance from the mean for the values both below and above the mean is 8. For **figure 1b,** this total distance is 9. The distances for the values below the mean are *balanced* by the distances for the values above the mean. This example shows why the mean value for a collection of numeric data indicates the balance point of the data distribution as well as the center of the distribution.

The two distributions shown in **figures 1a** and **1b** have the same mean. In comparing the two distributions, we can ask, "In which distribution do the data vary more from the mean?" One way to assess the degree of variation in data from the mean is to find the total distance between all data values and the mean. These distances are all considered to be positive, and their total is called the *sum of the absolute deviations* from the mean (SAD). For the data in **figure 1a**, the SAD is 16; for the data in **figure 1b**, the SAD is 18. Thus, according to the SAD, there is more variation in the data in **figure 1b** than in **figure 1a**.

The data in **figure 2a** differ more from the mean than do the data in **figure 2b**; however, the SAD for both sets of data is 8. The SAD in **figure 2b** is based on 8 data values but only 2 values in **figure 2a**. Dividing the SAD by the number of values gives the mean of the absolute deviations (MAD), which adjusts for the difference in group sizes: MAD = SAD/(number of values).

Fig. 2 Total points scored



The MAD for **figure 2a** is 4, whereas the MAD for **figure 2b** is 1. Thus, the data in **figure 2a** differ on average by 4 from the mean, whereas the data in **figure 2b** differ on average by 1 from the mean. The MAD provides a measure of average variation in the data compared with the mean and is a precursor for the standard deviation, the more commonly used measure for the degree variation in data from the mean. The standard deviation builds on the idea of looking at the distance from each data value to the mean. However, instead of averaging the absolute deviations, the squared deviations are averaged. This quantity, called the *variance*, is in squared units, and the standard deviation is the square root of the variance. It is recommended that the standard deviation be developed at level C. (See Kader 1998 for more information on this idea.)

MEMORIZING WORDS: A COMPARISON PROBLEM

Another useful way of summarizing center and spread in numerical data uses a five-number summary (minimum value, first quartile, median, third quartile, and maximum value). These markers form four groups with approximately 25 percent of the data in each group. The box plot, also called a box-and-whiskers plot, is a graphical representation of the data distribution based on the five-number summary. Together, they provide both visual and numerical information about—

- the *center* of the distribution, based on the location of the median within the central box of the graph;
- the *spread* in the middle 50 percent of the distribution, based on the width of the box (the interquartile range); and
- the overall spread of the distribution, based on the range.

Problems involving comparisons of two or more distributions are common in statistics, and box plots are especially useful in making such comparisons. Many states' mathematics curriculum guidelines call for box plots to be introduced in the middle grades.

The statistical task that follows was adapted from a lesson described in the Learning Math Project (WGBH Educational Foundation 2001). The task engages students in exploring how easy or hard it is to memorize words in lists. Data are summarized from two seventh-grade classes in Ohio. Both classes had completed a three-week unit on statistics that introduced students to box plots. The teacher launched the activity by asking students in small groups to describe situations in which they had to memorize some kind of text and the strategies they used. Included among the examples that students gave were studying, spelling, and playing video games. The teacher displayed the following two lists, and asked, "Which of the lists, A or B, do you think would be easier for most people to memorize?"

List A Words	**List B Nonwords**
BOSTON	MZAPDR
EAR	CTG
CART	OXCS
BUG	AEA
PAPER	SKEOC

The class conjectured that list A would be easier to memorize. They became intrigued with finding out whether they were correct and how different for other students was the experience of remembering the two lists.

Asking a Statistics Question

The class formulated the following question:

> Do people tend to score higher on list A than on list B?

Students then had to think about how to design a statistical investigation to study the question. The class discussion focused on how to *measure* someone's ability to recall words or nonwords. After an engaging discussion, the following experiment was agreed on:

- List A will be words, and list B will be nonwords. Each list will include twenty “words,” each with exactly three characters.
- Each student will be assigned a list and have two minutes to study it. After a thirty-second pause, each student will have two minutes to write down as many “words” from the list as he or she can recall.

Students discussed how to score the lists of “words” that were recalled, with several suggesting scoring methods that penalized students for listing words that were not on the original list. However, the final class decision was not to invoke a penalty.

Collecting Data to Address the Question

After a statistics question has been developed, data can be collected. Several students pointed out that some people are better at memorizing than others, and that it would be unfair if all those students ended up with the same list. They decided to form two balanced groups of students; each group would have some who were more proficient and others who were less proficient with memorization. Since those students are not known in advance, the class decided to randomly assign students to the two groups, hoping that randomness would provide two balanced groups. Introducing randomness in data collection is a transition for students from level A to level B. The use of randomness in data collection links probability to statistics and is more fully developed at level C. In the companion curriculum article in the *Mathematics Teacher*, Scheaffer and Tabor (2008) discuss the use of randomness in data collection and its role in statistical inference.

Fig. 3 Comparative dot plots for scores

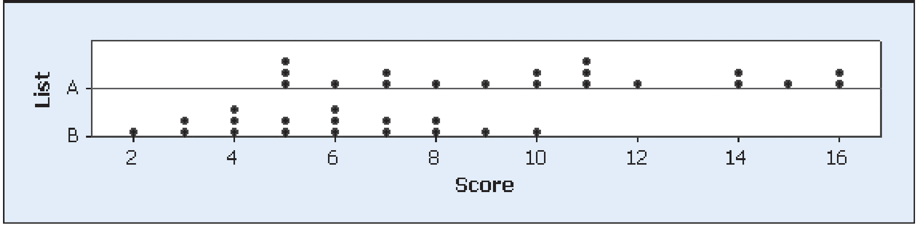


Table 1 Questions related to dot plots and sample student responses

Question Number	Question	Sample Student Responses
1	Looking at the dot plots, did all students who studied list A score higher than all students who studied list B? Explain.	No, because list B had numbers 5–10 and in list A, people had also scored 5–10. No, only nine people in list A scored higher than the highest score in list B.
2	Looking at the dot plots, did most students who studied list A score higher than students who studied list B? Explain.	Yes, most did better, but not all of them. About half of them scored better than them, 47 percent. No, because only 47 percent scored above the highest score on list B.
3	Looking at the dot plots, give three statements (based on the plots) that suggest that students who studied list A generally scored higher than students who studied list B.	Minimum of list B was 2. Minimum of list A was 5. List A did not go from 2 to 4 but B has 35 percent from 2–4. Forty-seven percent scored higher than list B; 53 percent of list A were from 5–10.
4	In what ways are the dot plots different? Explain how they are different.	List A is more spread out while list B is more clumped together.

In the two classes, nineteen students were assigned to list A and seventeen to list B. After separating into the two groups, the students carried out the procedure described earlier. The results were collected, scored by the teacher, and presented to the classes the next day.

Part 1: Analysis and Interpretation

In the initial analysis of the data, students constructed comparative dot plots by hand (**fig. 3** shows a computer-generated plot). Students’ comments during discussion of the dot plots included these:

Joey: Forty-seven percent of the list A kids did better than the 10.

Samantha: The data is clustered and shifted more high on graph A.

Josh: The minimum and maximum on graph A are higher than on B.

Fig. 4 Five-number summaries and comparative box plots for scores

Summary	List A (n = 19)	List B (n = 17)
Minimum	5	2
First quartile	7	4
Median	10	6
Third quartile	14	7
Maximum	16	10

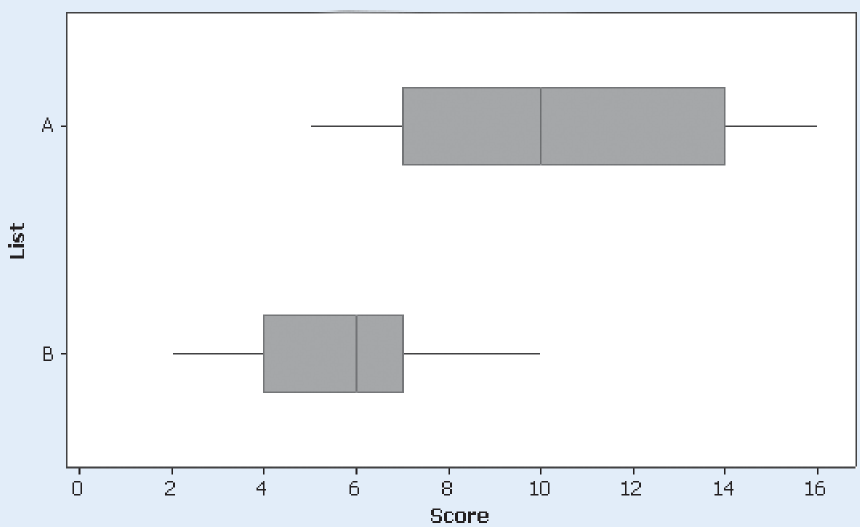


Table 2 Questions related to box plots and sample student responses

Question Number	Question	Sample Student Responses
1	Looking at the box plots and five-number summaries, did all students who studied list A do better than all students who studied list B? Explain.	No, because if you put both box plots on top of each other they would overlap, meaning somebody from both box plots scored the same. No, because list B goes up to the median on list A, which means half of the students had done the same as some of the students on list B.
2	Looking at the box plots and five-number summaries, give three statements (based on the plots) that suggest that students who studied list A did better	List A's median is higher than list B's median by 40 percent. About 50 percent on list A did better than the highest score on B.
3	In what ways are the box plots different? Explain how they are different.	List A has a larger box than list B's.

After the discussion, the teacher posed several specific questions based on the dot plots. Students worked in groups to create arguments about whether people were more successful when working with list A compared with list B. The questions and sample student responses are shown in **table 1**.

Student responses to question 1 were generally very good. They recognized that although students generally did better on list A, there is some overlap in the dot plots. Responses to question 2 also were quite good. Many students identified that 9 out of 19 (47 percent) of scores on list A are above the highest score on list B. Questions 3 and 4 were less specific in nature, and students found them to be more challenging. The responses varied more. In question 4, many students recognized the greater variation in list A, but no students compared the ranges.

Part 2: Analysis and Interpretation

Next, the teacher asked the students to determine the five-number summary for each list and to construct comparative box plots by hand (a computer-generated plot is shown in **fig. 4**). They returned to the statistical question they were trying to answer.

> Do people tend to score higher on list A than on list B?

The groups responded to three questions that were based on the box plot analysis. The questions and the sample student responses are shown in **table 2**.

Students generally had more difficulties addressing the questions with box plots than they did with dot plots. As with the dot plots, the goal in question 1 is to recognize that there is some overlap in the box plots. For question 2, many students recognized that the median for list A (10) is higher than the median for list B (6). Many students also used statements involving proportional reasoning in addressing question 2. Several students compared quartiles and extremes and pointed out that these summaries are all higher for list A. On question 3, a few students recognized that the box plots suggest more variation in the scores from list A than from list B. Ideally, students would support this observation by comparing and contrasting summary measures of variation, in this case, the ranges and the interquartile ranges. As students gain experience with statistical problem solving, the goal is for them to become adept at connecting numerical summary measures to graphical representations of data.

Students and the teacher were extremely positive about this statistical investigation. The activity allowed students to be involved with all four components of the statistical problem-solving process. During the activity, as students discussed what they saw in the plots, they used statistical terms such as *clusters, spread, minimum, maximum, median, most data*, and *percentages*. By working together, they were able to feed off one another's statistical ideas. This activity illustrates how authentic problems can be used to develop statistical concepts. Such activities take students beyond simply calculating summary measures and constructing plots and push them to think about how to use the summary measures and plots to address various questions.

Although the box plot is a relatively simple graph to construct, Bakker, Biehler, and Konold (2004) point out that middle-grades students generally have difficulties interpreting box plots. One explanation is that the four groups of data formed from the five-number summary are based on percentages and that their interpretation requires proportional reasoning. Guidance and practice are required

As students gain experience with statistical problem solving, the goal is for them to become adept at connecting numerical summary measures to graphical representations of data

for middle-grades students to understand the five-number summary and be able to interpret and compare box plots. Middle school is an appropriate place for this effort to begin, because developing proportional reasoning skills is a goal for the middle and junior high grades.

SUMMARY

The GAISE report describes three developmental levels of evolving statistical concepts. Experiences at level B should provide the link between the more concrete experiences from level A to the inferential types of thinking developed at level C. Two contexts for developing student thinking about the center and spread for discrete numerical data are explored. The representations discussed in these activities also are appropriate for continuous numerical data. These statistical tasks promote student understanding of various numerical summaries and illustrate connections between various representations. Both tasks build on ideas developed at level A, promote proportional reasoning at level B, and provide the foundation for statistical thinking developed at level C.

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