Calculating Measures of Dispersion

Reporting Category	Statistics
Topic	Calculating Mean Absolute Deviation, Variance, and Standard Deviation
Primary SOL	A.9 The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.
Related SOL	

Materials

- Graphing Calculators
- Overhead or computer graphing calculator to be projected to the class (optional)

Vocabulary

mean, median, mode, range, measure of central tendency
dispersion, mean absolute deviation, standard deviation, summation notation, variance (A.9)

Student/Teacher Actions – What should students be doing? What should teachers be doing to facilitate learning?

1. Present students with the following scenario:
   Students in Mrs. Smith's Communication Club are interested in how many hours they spend watching television. They collected the following data which shows the number of hours they each watched television in one week.
   3, 8.5, 9, 9, 12.5, 14, 16.5, 18, 19, 20.5
2. Ask students to think about what they could do with this data. Have them work in small groups or with a partner to do what they can with the data. Some will represent it graphically, find the mean, median, mode, range, etc.
3. Ask students to share what they chose to do with the data. Question the students about what the graph or statistic says about the information after each idea is shared. Ex. What does the mean tell us about the data? What does this histogram show us about the data?
4. In the discussion attach the vocabulary of mean, median and mode being measures of central tendency and range being a measure of dispersion or the spread of the data.
5. Tell students another way to measure dispersion is to compare the elements to the mean. Ask students to help you find the deviation of each element from the mean , or the distance each element in the data is from the mean. Ask students how they would find this deviation. (They should subtract the mean from each value.) Create a chart for all students to see with the original elements on one side and a space for the deviation

from the mean on the other. Record the mean beside the chart, using and introducing the symbol of μ for the mean.

x	$x-\mu$
3	
8.5	
9	
9	
12.5	
14	
16.5	
18	
19	
20.5	

6. Have the class help you fill in the chart. Ask them to calculate the average distance from the mean or average deviation. Ask for their observations (the deviations sum will be zero). Ask students if this will always occur and why or why not. Once students determine that this will always occur, because of the negative values ask students how they could ensure that a number is always positive. Continue the discussion until the idea of absolute value emerges.
7. Have students find the sum of deviations using the absolute values. Introduce students to the name for this descriptive statistic as the mean absolute deviation. Ask students to summarize how they found the mean absolute deviation. Present students with the formula below and have them discuss how it matches what they did to calculate the mean absolute deviation and their summary of the process. If you have not previously discussed summation notation, you will need to explain this notation.

Examples of summation notation

$$\sum_{i=1}^{5} i = 1+2+3+4+5 \qquad \sum_{i=1}^{4} x_i = x_1 + x_2 + x_3 + x_4$$

Mean absolute deviation $= \dfrac{\sum_{i=1}^{n} |x_i - \mu|}{n}$, where μ represents the mean of the data set, n represents the number of elements in the data set, and x_i represents the i^{th} element of the data set.

8. Ask students what the mean absolute deviation could tell them about the data. What would a high mean absolute deviation indicate? What about a low value for the mean absolute deviation?
9. Tell students that there is another method for finding the dispersion about the mean. Guide students to think of other ways to not have negatives in a set of numbers. You may ask them questions to get students thinking about their past experiences with quadratics and squaring numbers.
10. Have students find the squared deviations (you may want to add another column to the chart for this). Then have them find the average of the squared deviations.
11. Introduce this value as the variance, another measure of dispersion. Give the symbol for this value σ^2. Also show the formula for finding the variance and have students explain how the formula correlates to what they did to compute the variance.

Variance $\left(\sigma^2\right) = \dfrac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$, where μ represents the mean of the data set, n represents the number of elements in the data set, and x_i represents the i^{th} element of the data set.

12. Ask students what the unit of the variance is (hours^2). Ask how they could get the unit to be the same as the units in the data (by taking the square root). Explain that this is called the standard deviation (σ) and is another measure of dispersion. Provide students with the formula and have them explain how this formula addresses dispersion.

Standard deviation $(\sigma) = \sqrt{\dfrac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}}$, where μ represents the mean of the data set, n represents the number of elements in the data set, and x_i represents the i^{th} element of the data set.

13. Ask students what the standard deviation could tell them about the data. What would a high standard deviation indicate? What about a low value for the standard deviation?
14. Using the data from this problem, instruct students on how to calculate the mean absolute deviation, variance, and standard deviation in their graphing calculators. Graphing calculator keystrokes can be found in the Technical Assistance Document for A.9 found on the VDOE Website—Instruction > Mathematics > High School Mathematics Instructional Resources. The instructions include another set of data which may be given to students for additional practice. You may also need to address the difference in the calculator using $\overline{X}$ instead of μ to represent the arithmetic mean.

Assessment

- **Questions**
  - What are some ways you can measure the dispersion of a set of data?
  - Explain how to calculate each of these measures without a calculator.
- **Journal/writing prompts**
  - Explain why the methods of mean absolute deviation, variance, and standard deviation are used to measure dispersion.
  - Compare and contrast standard deviation and mean absolute deviation.
- **Other**
  - Have students collect data from their class and compute standard deviation and mean absolute deviation. (Use the graphing calculator if the data set is large. If calculating by hand keep the number of data elements less than 10.)

Extensions and Connections (for all students)

- Have students remove the outlier in the data and recomputed the standard deviation and mean absolute deviation. Which seems to be more affected by an outlier?

Strategies for Differentiation

- Have students create a visual diagram to compare and contrast standard deviation and absolute mean deviation.
- Have students create a graphic organizer to arrange the vocabulary, statistics notation, and steps for calculating measures of dispersion.
- Give students with visual discrimination/processing issues a sheet that includes the formulas and definitions in the gray boxes and example problems for each with data and correct numbers plugged into the formulas.
- Provide warm-up problems (prior to the lesson) that include symbols used in the various statistics formulas to have students become familiar with the symbolic language. For example,

- Evaluate the following if $x = 6,\ y = 2,\ z = 4$.
  - $(x - y)^2$
  - $\sqrt{\frac{(x - y)^2}{z}}$
  - $\frac{x - y}{z}$
- Evaluate the following if $x = 9,\ \mu = 4,\ \sigma = 4$.
  - $(x_1 - \mu)^2$
  - $\frac{x_1 - \mu}{\sigma}$
  - $\sqrt{\frac{x_1 - \mu}{\sigma}}$