z-Scores

Reporting Category	Statistics
Topic	z-Scores
Primary SOL	A.9 The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.
Related SOL	

Materials

- Graphing Calculators
- Handouts

Vocabulary

mean

dispersion, standard deviation, z-score (A.9)

Student/Teacher Actions – What should students be doing? What should teachers be doing to facilitate learning?

1. Review standard deviation by asking students: What is standard deviation? How can you find it? What does it measure? When might you use it?
2. Have students complete the Heights of Basketball Players handout in small groups or partners. You may need to review how to construct a line plot if your students are not familiar.
3. After students have completed the handout, bring the class together and discuss the solutions. Focus the discussion on the explanations of finding out how many elements were within one standard deviation of the mean and what percent of the data was more than one standard deviation **above** the mean. As you discuss the final question, some students may have just stated the element of 67 inches was more than one standard deviation below the mean. If no one found the exact number of standard deviations, ask questions probing students to find a more precise answer. It is fine if the z-score is a positive value at this point.
4. Explain to students that a z-score gives you the number of standard deviations an element is from the mean. Give them the following formula and have them explain how the formula finds the precise number of standard deviations from the mean.

$$\text{z-score}(z) = \frac{x - \mu}{\sigma}$$, where x represents an element of the data set, μ represents the mean of the data set, and σ represents the standard deviation of the data set.

5. Have students calculate the z-score for the elements 69 and 78 using the formula.
6. Ask students why one z-score is positive and one is negative. If students used a positive z-score for the last question on the handout, go back and address this now.
7. Have students look at the data and determine how many z-scores would be positive and how many would be negative. Have them explain why this makes sense.
8. Ask students if they did not know an element in the data, but they did know the z-score could they figure out the element. What additional information would they need?

Assessment

- **Questions**
  - What do z-scores tell you about the position of elements with respect to the mean?
  - Why are some z-scores negative?
  - An element in the set has a z-score of 1.25, explain how you would find the value of that element. Does this z-score represent an element in the given data set?
- **Journal/writing prompts**
  - Given the standard deviation, the mean, and the value of an element of the data set, explain how you would find the associated z-score.
  - How do z-scores relate to their associated element's value?
- **Other**
  - Have students find z-scores of other data sets they have worked with.

Extensions and Connections (for all students)

- Give students a mean and standard deviation of a data set. Also, give them a z-score and ask them to find element associated with the z-score.

Strategies for Differentiation

- Have students create a graphic organizer to summarize the important ideas in calculating and using z-scores
- On the Heights of Basketball Players handout have students label the mean and z-scores for each of the values on their line plot. This will help them to see that z-scores to the left of the mean are negative and those to the right of the mean are positive. You can relate this to their prior knowledge of a number line, in this case the mean is representing zero.

- Once the line plot is made and standard deviation is computed, have the students measure one standard deviation using the same scale as their line plot. Students can cut the standard deviation out of an index card or sticky note. Students can then physically measure the number of standard deviations as well as mathematically computing it.

Heights of Basketball Players

The following are the heights (in inches) of the Greenwood High basketball team members.

72, 66, 67, 69, 73, 74, 72, 71, 75, 69, 72, 70, 78, 70, 71, 73

1. Construct a line plot of the data.

2. Calculate the mean and standard deviation of this data set.

3. How many elements are above the mean? How many are below?

4. How many elements are within one standard deviation of the mean? Explain how you arrived at your solution.

5. What percent of the data is more than one standard deviation above the mean?

6. How many standard deviations from the mean is the player with a height of 67 inches? Explain how you arrived at your solution.