Rational Functions: Intercepts, Asymptotes and Discontinuity

Reporting Category Functions
Topic Asymptotes, discontinuity
Primary SOL AII.7 – The student will investigate and analyze functions algebraically and graphically. Key concepts include a) domain and range, including limited and discontinuous domains and ranges; b) zeros; c) x- and y-intercepts; and e) asymptotes.
Related SOL AII.7 d, f (increasing/decreasing, end behavior), AII.1d (factoring)

Materials

- Graphing calculator
- “Investigating Functions” handout
- “What’s My Function?” handout and cutouts for the board
- Activity cards for Day 2 activity

Vocabulary

x-intercept, y-intercept, domain, range, zeros (earlier grades)
discontinuous domain, discontinuous range, point of discontinuity, asymptote (AII.7)

Student/Teacher Actions

1. (Day 1) Do a quick review of factoring. Include factoring out a common factor, the difference of two squares, and a trinomial with a leading coefficient greater than one.
2. Have students graph (on a calculator) $f(x)=\frac{x+2}{x+3}$. Students discuss the graph with a partner and write down anything they can about the function.
3. Share thoughts with the whole group.
4. Have students complete a table of values for the function by hand. They should use the trends of the graph to lead them to their chosen domain values. After a couple of minutes, if students are not already doing so, have them choose domain values that are smaller than -50 and greater than 50, and values surrounding, and including, -3. Next have them see what the value of the function is when x is 100 and -100, and when x is -2.999 and -3.001.
5. Introduce the term, ‘asymptote’. If you are teaching this after end behavior and increasing/decreasing intervals, talk about the behaviors surrounding the asymptote in

those terms. Say " A *horizontal asymptote* of a rational function represents *end behavior*."

6. Have students graph $f(x)=\dfrac{x^2-16}{x-4}$ on their calculators. Next, have them simplify the rational expression on paper. Without deleting the first function from the calculator, graph a second function, $y=x+4$. Discuss why it makes sense that it would be the same line. Then discuss why it cannot be the same line. Once students have realized that x cannot be 4 in the original function, have them clear the second function off of the calculator and take a closer look at the rational function. Students can zoom in, trace the line and choose an x-value of 4, or look at a table to discover that there is actually a 'hole' in the graph. Have students consider why x = 4 is not in the domain of *f.*

7. Introduce 'discontinuous functions', 'discontinuous domain' and 'point of discontinuity'.

8. Now graph $f(x)=\dfrac{x-4}{x^2-16}$. Factor the denominator, and discover what values of x will make the function undefined. Pairs should write down everything they notice about this graph. Hopefully they will discover there is a horizontal asymptote (y=0), as well as a vertical asymptote (x=-4). Be sure they realize there is discontinuity where x=4. Trace the graph with your finger, exaggerating the point of discontinuity.

9. In groups, students graph the following functions:

   a) $f(x)=\dfrac{x}{x^2}$ b) $f(x)=\dfrac{x}{x+1}$ c) $f(x)=\dfrac{3x}{x+1}$ d) $f(x)=\dfrac{x^2}{x+1}$

10. Students compare and contrast the functions algebraically and graphically. Challenge them to make a conclusion about horizontal asymptotes based on the algebraic form of the function.

    (degree of numerator > degree of denominator $\Rightarrow$ no horizontal asymptote)
    (degree of numerator < degree of denominator $\Rightarrow$ horizontal asymptote at y = 0)
    (degree of numerator = degree of denominator $\Rightarrow$ horizontal asymptote at

    $$y=\frac{\textit{coefficient of highest degree term in numerator}}{\textit{coefficient of highest degreee term in denominator}})$$

11. Look back at all of the functions you graphed and determine the x- and y-intercepts. Stress that the zeros of the function are the x-intercepts, and can often be easily found once a numerator is factored.

12. Students receive “Investigating Functions” handout and do #1-6 with a partner. Students complete the worksheet *without* a calculator. As two sets of partners complete the worksheet, have the pairs compare answers and discuss differences. Once all pairs have checked and worked through questions with another pair, have the foursome group up with another group of four to compare.

13. Students may now graph the functions on the calculator and check themselves.

14. **GAME:** Give students “What’s my Function?” worksheet. Students compete to see who can write the most functions correctly (algebraically). Calculators may not be used! Tape the graphs on the board (masters included). Once students have completed the handout, students write their responses with their initials underneath the corresponding graph. (There will be several functions written underneath each graph.) Once students have completed this task, have them graph the student responses on their calculators to determine which functions are correct. The student with the most correct wins a prize. (Depending on your class, you may want to make this a partner or larger group competition.)

15. (Day 2) Extend the discussion of asymptotes and intercepts to include exponential functions. Have students complete #7-12 on the “Investigating Functions” handout from yesterday. If you have already taught end behavior and domain and range, students may complete the ‘extension’ exercise following the problem set.

16. ACTIVITY: **1-**Place function cards across the top of the board. **2**-For each function, write each of ‘x-intercepts, y-intercepts, horizontal asymptotes, vertical asymptotes and points of discontinuity’ on separate lines below the function. **3**-Hand out a card to each student. If there are extra cards, some students will receive more than one. **4**- Have students place their cards in the appropriate space. (It is suggested to copy masters on colored paper and laminate them. Adhesive magnets are great for placing the cards on white boards, but tape works fine, too.)

Assessment

- **Questions**
    - What are all asymptotes and points of discontinuity for $f(x)=\dfrac{x^2-5x-14}{2x^2-8}$?
    - Without a calculator, determine the x- and y-intercepts of $f(x)=(-2)^x+3$.

- **Journal/writing prompts**
    - Describe how, without graphing, you can determine whether or not a rational function has any horizontal asymptotes and what the horizontal asymptotes are.
    - Explain how simplifying a rational function can help you determine any vertical asymptotes or points of discontinuity for the function.

Extensions and Connections (for all students)

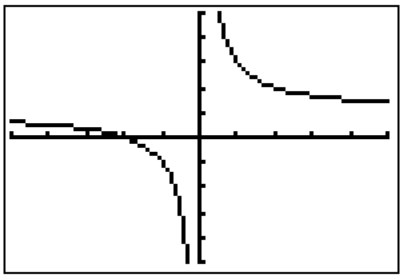
- Have students draw their own graphs with vertical and/or horizontal asymptotes and give to a classmate to write the algebraic function that is graphed.
- Discuss how transformations on the functions would affect location of asymptotes.

Strategies for Differentiation

- Play a matching game, either with cards or an interactive white board, which matches functions to their graphs.
- Use pictorial representations to reinforce vocabulary.
- Look for a website that demonstrates the relationship between a function and it's asymptotes, discontinuity and intercepts.
- Make extended use of graphing calculator and/or graphing calculator software to reinforce characteristics of a rational function.

What's My Function?

Directions: Write the algebraic function for each graph shown.

1. 

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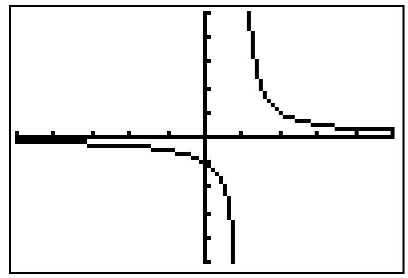
2. 

x = -4 y =

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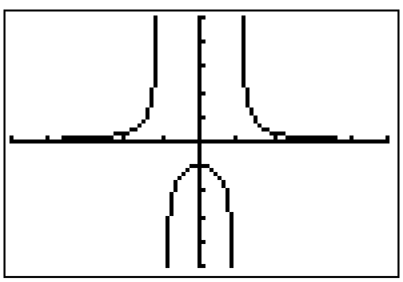
3. 

x = 0 y =

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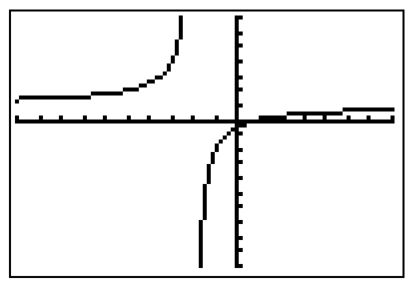
4. 

x = 1 y = 0

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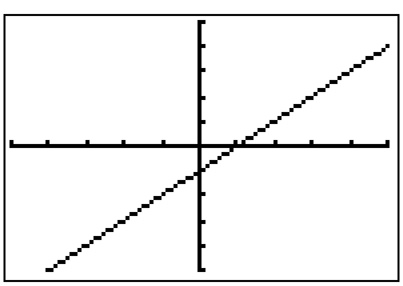
5. 

x = -1 y =

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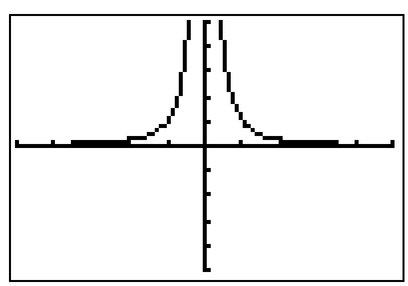
6. 

(Challenge)

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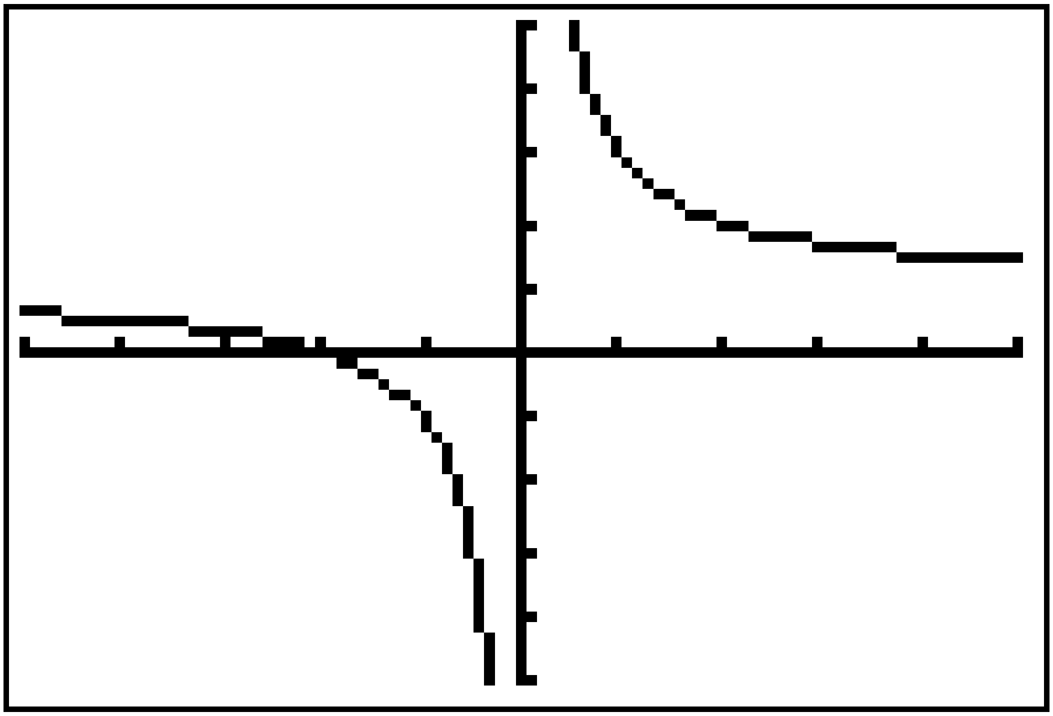
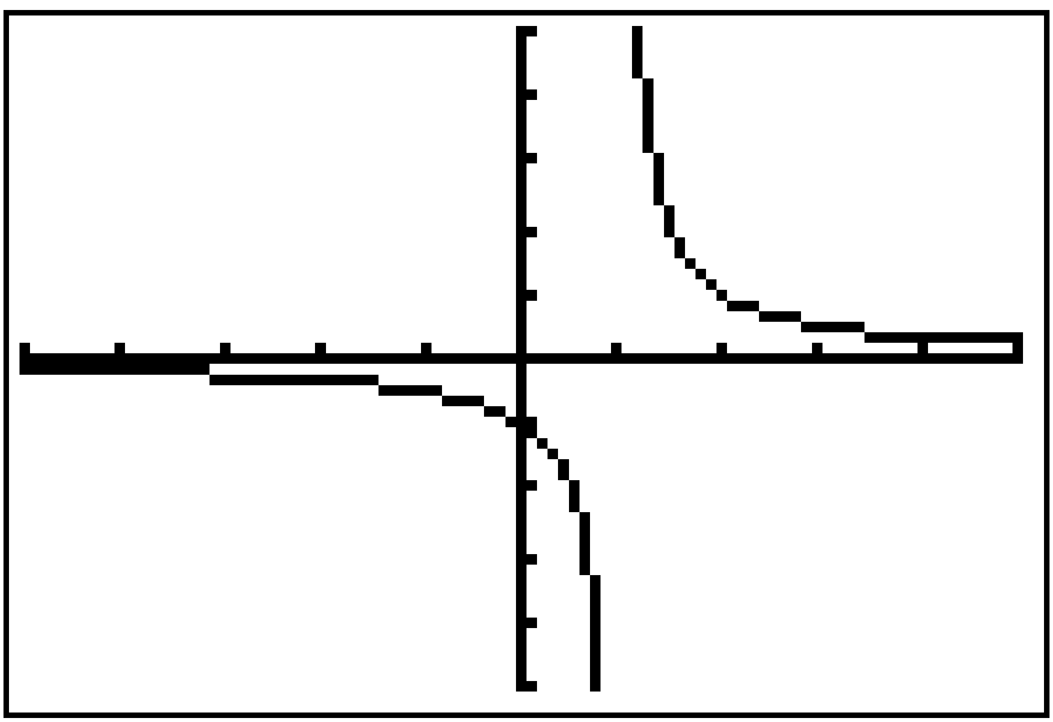
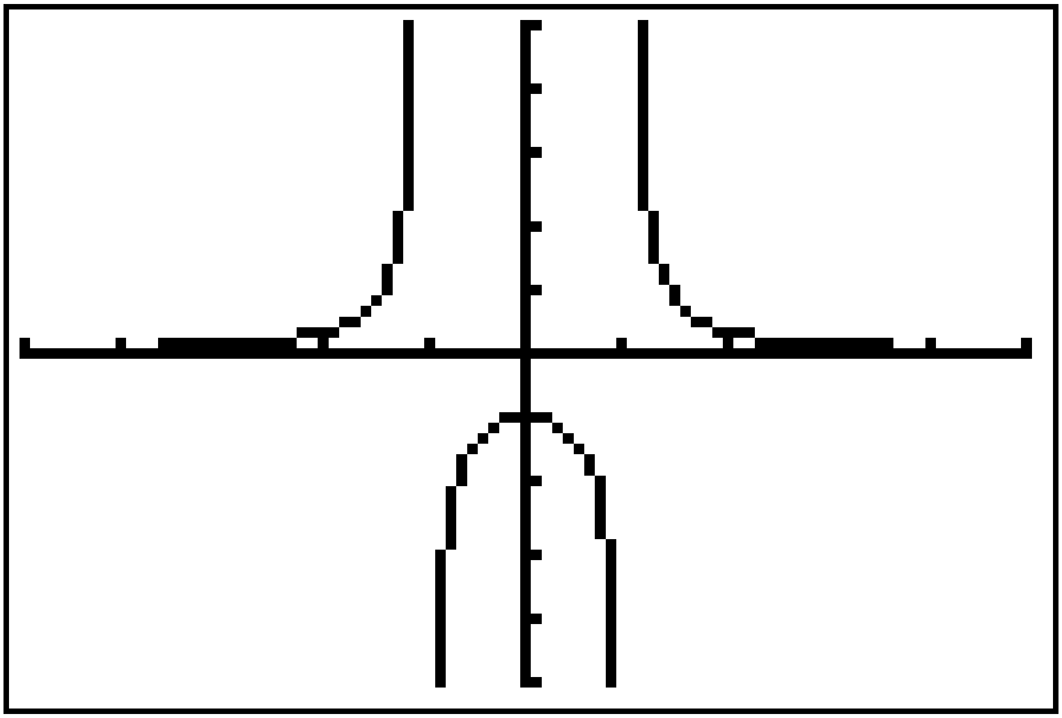
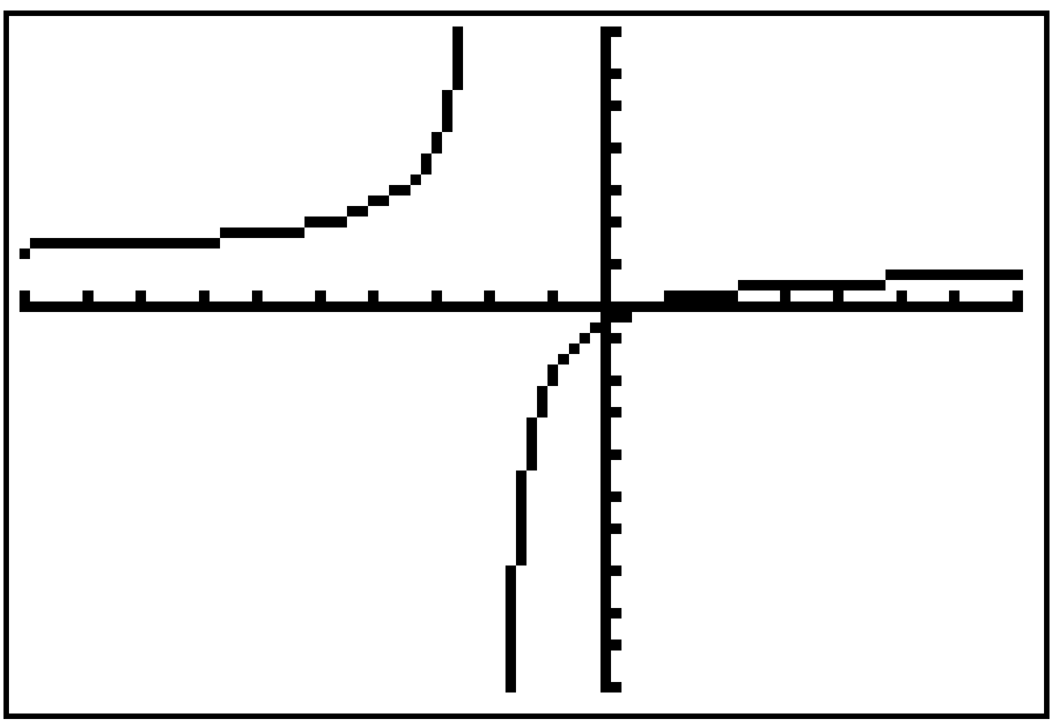




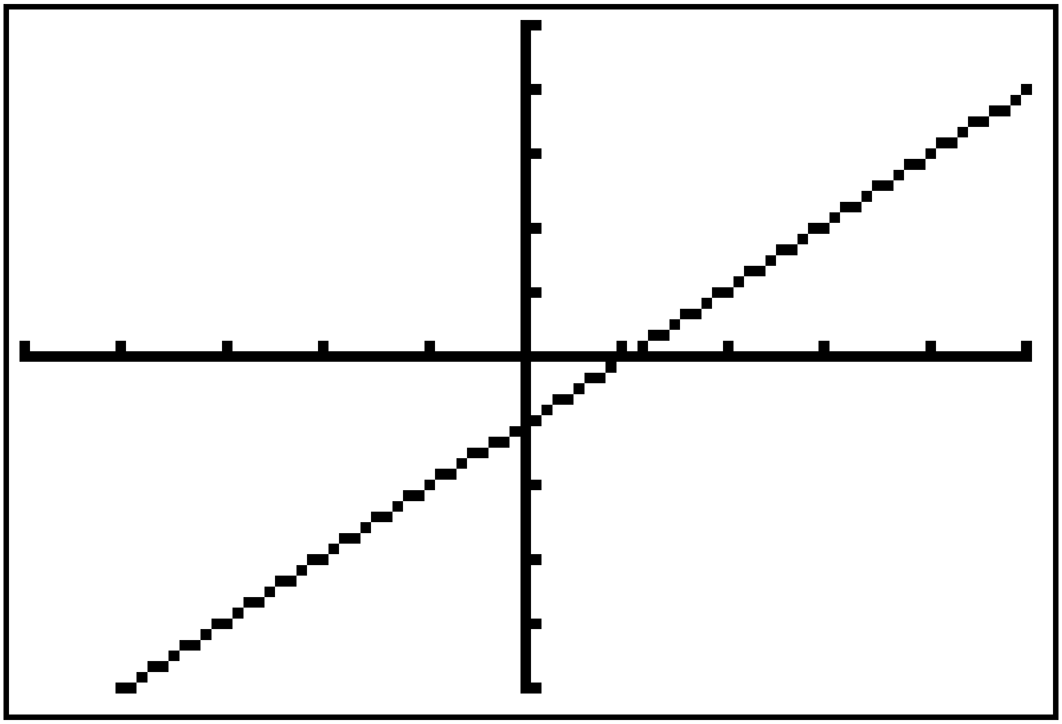
x = -4 y =



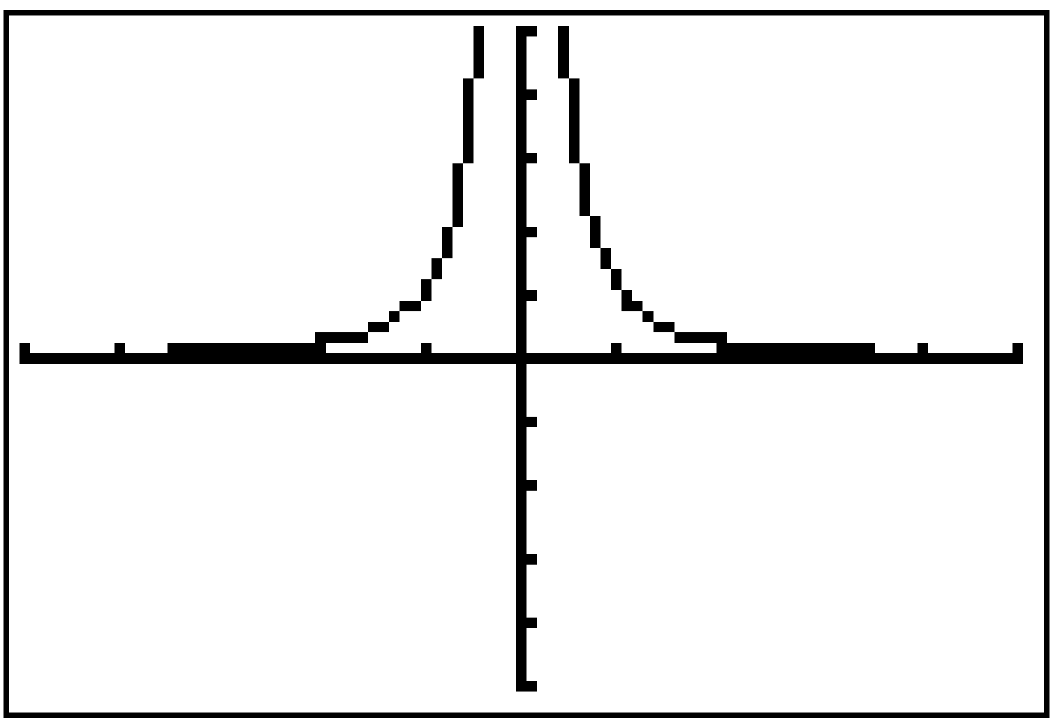
x = 0 y =



x = 1 y = 0



x = -1 y =



Challenge

Investigating Functions

Determine the following for each function: x- and y- intercepts, discontinuous points (disc. Pts.) , and vertical and horizontal asymptotes (h.a. and v.a.)

1) $f(x) = \dfrac{x+2}{x+3}$

- x- & y-int.________________________
- disc. pts.________________________
- v. a. & h.a.________________________

2) $f(x) = \dfrac{x^2-9}{x-3}$

- x- & y-int.________________________
- disc. pts.________________________
- v. a. & h.a.________________________

3) $f(x) = \dfrac{2x^2-5x-12}{x^2-16}$

- x- & y-int.________________________
- disc. pts.________________________
- v. a. & h.a. ________________________

4) $f(x) = \dfrac{5}{x-6}$

- x- & y-int.________________________
- disc. pts.________________________
- v. a. & h.a. ________________________

5) $f(x) = \frac{x+2}{x^2-4}$

- x- & y-int.__________________________
- disc. pts.___________________________
- v. a. & h.a. _________________________

6) $f(x) = \frac{1}{x}$

- x- & y-int.__________________________
- disc. pts.___________________________
- v. a. & h.a. _________________________

7) $y = 2^x$

- x- & y-int.__________________________
- disc. pts.___________________________
- v. a. & h.a. _________________________

8) $y = 2^{x+2}$

- x- & y-int.__________________________
- disc. pts.___________________________
- v. a. & h.a. _________________________

9) $y = -(2)^{x-2}$

- x- & y-int.__________________________
- disc. pts.___________________________
- v. a. & h.a. _________________________

10) $3xy = 6$

- x- & y-int.__________________________
- disc. pts.___________________________
- v. a. & h.a. _________________________

11) $y = \frac{3}{x}$

- x- & y-int.________________________
- disc. pts.________________________
- v. a. & h.a. ________________________

12) $-2xy = 1$

- x- & y-int.________________________
- disc. pts.________________________
- v. a. & h.a. ________________________

Extension: On your calculator, graph each of the twelve functions individually. State the domain and range of each function and describe the end behavior.

Rational Functions

(Card Activity)

$$y = \frac{x+3}{x+1}$$

$$y = \frac{x+3}{x^2}$$

$$y = \frac{3x^2 - 5x - 12}{x - 3}$$

$$y = \frac{x^2 - x - 12}{2x^2 - 32}$$

$$5xy = 10$$

$$y = 2^x$$

none	$x = 0$
none	$y = 0$

none	none
(0,1)	$y = 0$

none

none

$(-3,0)$ $(0,3)$

$x = -1$ $y = 1$

none	$y = 0$
$x = 0$	$(-3,0)$

(0,-3)	(0,4)
none	none

where	
$x = 3$	$(-4/3, 0)$
none	$y = 1/2$

$x = -4$ $(-3,0)$

$(0,3/8)$ where $x = 4$