

Functions: Domain, Range, End behavior, Increasing or Decreasing

Reporting Category	Functions
Topic	Finding domain and range and determining where a function is increasing or decreasing.
Primary SOL	All.7 - The student will investigate and analyze functions algebraically and graphically. Key concepts will include: a) domain and range, d) intervals in which a function is increasing or decreasing; and f) end behavior
Related SOL	All.6 (Function families)

Materials

- Graphing calculators
- “Where to Begin and End” handout
- “Domain and Range” handout

Vocabulary

domain, range (earlier grades)

increasing function, decreasing function, end behavior (All.7)

Student/Teacher Actions

1. Have students graph the function $f(x) = x^2$, while you graph it so that it is projected onto the board or wall.
2. Have students discuss with a partner the definitions of domain and range, and determine what the domain and range of the quadratic function are.
3. Discuss the difference between a continuous function and a discontinuous function and have students determine whether or not the quadratic is a continuous function. (In the next lesson, we will be looking closely at discontinuous functions.)
4. While tracing the graph with your finger, ask the students to tell you when the function is increasing and when it is decreasing. Ask “Exactly where did the graph change from decreasing to increasing?”
5. Discuss the end behavior of the function, both as x approaches negative infinity and positive infinity.

6. Demonstrate, and have students copy into notes, how to express the domain $\{x \mid x \in \mathfrak{R}\}$, the range $\{f(x) \mid f(x) \geq 0\}$, intervals where the function is decreasing $\{x \mid -\infty < x < 0\}$ and increasing $\{x \mid 0 < x < \infty\}$, and end behavior (as x approaches negative infinity, $f(x)$ approaches positive infinity; as x approaches positive infinity, $f(x)$ approaches positive infinity.)
7. Graph on the calculator, with the students, $f(x) = \sqrt{2x - 6}$.
8. Ask, "Why do we see the graph in the first quadrant only? How would the domain and range, as well as the end behavior, be different if we were looking at a parabola in the first and fourth quadrants?" (Draw what you are referring to on the board.) Erase it and refer to the original function again.
9. Students express domain, range, intervals for increasing and decreasing, and end behavior of this function on their paper. Call volunteers to write one each on the board. Discuss the fact that we cannot describe end behavior as x approaches negative infinity since x cannot be less than 3.
10. Students complete "Where to Begin and End" handout in pairs or groups.
11. Next day review: "Domain and Range" handout

Assessment

- **Questions**
 - State the domain and range for $f(x) = \sqrt{x - 4}$.
 - Describe the end behavior of $f(x) = 2^x$.
- **Journal/writing prompts**
 - Create and describe a function, both algebraically and graphically, that would be increasing in two different intervals and decreasing in one interval.
 - Compare and contrast the end behaviors of a quadratic function and its reflection over the x -axis.

Extensions and Connections (for all students)

- State the domain and range for a circle with center (2,5) and radius 4.
- Demonstrate the use of interval notation to define intervals of increasing and decreasing.

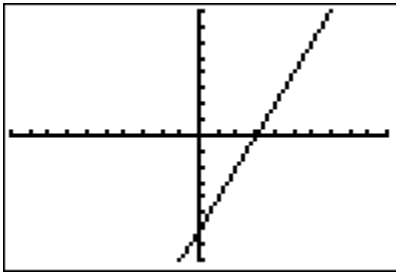
Strategies for Differentiation

- Have students model with their arms end behavior of polynomial functions, depending on the sign of the leading coefficient and the degree of the polynomial.
- Use an interactive white board with graphing software that allows as student to manipulate graphs.

Where to Begin and End

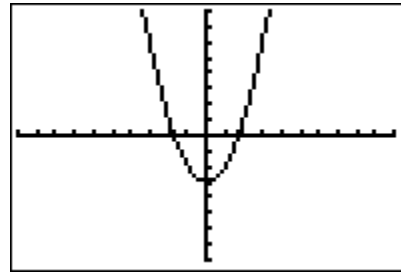
For each function, state the domain and range, name the intervals where the function is increasing or decreasing, and describe the end behavior.

1.



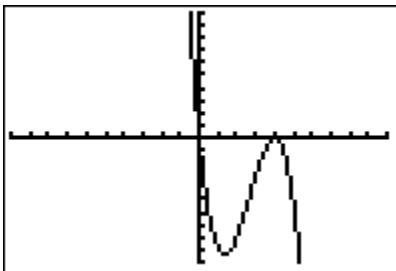
- Domain _____
 - Range _____
 - Increasing _____
 - Decreasing _____
 - End Behavior _____
- _____

2.



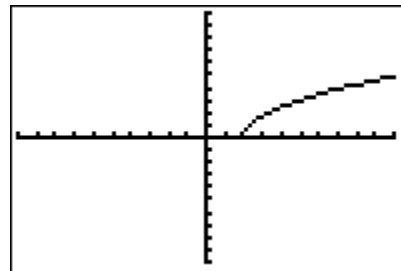
- Domain _____
 - Range _____
 - Increasing _____
 - Decreasing _____
 - End Behavior _____
- _____

3.



- Domain _____
 - Range _____
 - Increasing _____
 - Decreasing _____
 - End Behavior _____
- _____

4.



- Domain _____
 - Range _____
 - Increasing _____
 - Decreasing _____
 - End Behavior _____
- _____

5. $f(x) = 3x + 5$

- Domain _____
 - Range _____
 - Increasing _____
 - Decreasing _____
 - End Behavior _____
- _____

6. $f(x) = -3x + 5$

- Domain _____
 - Range _____
 - Increasing _____
 - Decreasing _____
 - End Behavior _____
- _____

7. $f(x) = x^2$

- Domain _____
 - Range _____
 - Increasing _____
 - Decreasing _____
 - End Behavior _____
- _____

8. $f(x) = (x + 3)^2$

- Domain _____
 - Range _____
 - Increasing _____
 - Decreasing _____
 - End Behavior _____
- _____

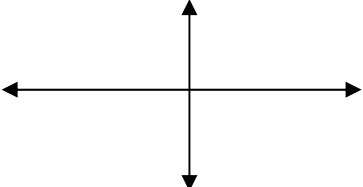
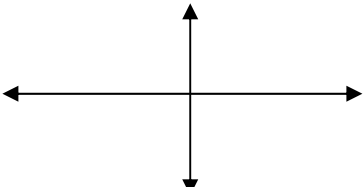
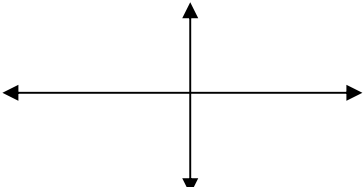
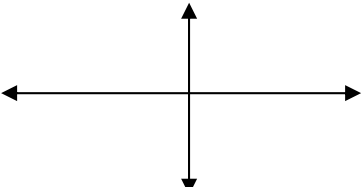
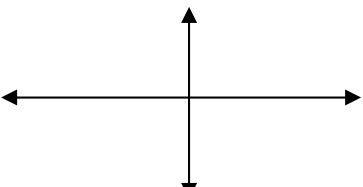
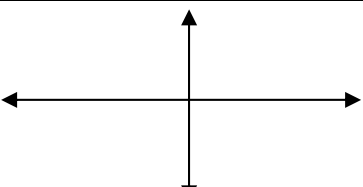
9. $f(x) = -2x^2 - 2$

- Domain _____
 - Range _____
 - Increasing _____
 - Decreasing _____
 - End Behavior _____
- _____

10. $f(x) = x^3 + 6x^2 + 9x$

- Domain _____
 - Range _____
 - Increasing _____
 - Decreasing _____
 - End Behavior _____
- _____

Domain and Range

Function	Graph	Domain and Range
1. $f(x) = \frac{1}{2}x^2$		Domain: Range:
2. $y = x^2 + 3$		Domain: Range:
3. $y = -3x^2$		Domain: Range:
4. $y = x(5 - x)$		Domain: Range:
5. $m(x) = \left(\frac{1}{3}\right)^x$		Domain: Range:
6. $h(x) = 3^x$		Domain: Range:

Describe the end behavior and name the intervals where each function is increasing or decreasing.