Normal Distributions

Reporting Category	Functions and Statistics
Topic	Analyzing and using Normal Curve
Primary SOL	AII.11 - The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.
Related SOL	AI.9 (standard deviation, z-scores)

Materials

- Graphing Calculator
- Normal Curve Graph Paper
- Standard Normal Probabilities table (z-score table)

Vocabulary

Mean, median, mode, standard deviation, z-score, probability, quartile (earlier grades)
Normal distribution, normal curve, percentile, area under a curve, probability density function, discrete vs. continuous data (AII.11)

Student/Teacher Actions

- Connecting to prior knowledge – Have students individually or in pairs complete the *Statistics Review* and discuss the responses as a class. This activity reviews critical concepts necessary for understanding and interpreting normal distributions and problems associated with them, so a thorough discussion of the results is important.
- *Exploration#1* connects discrete and a histogram with the normal curve. Emphasis should be placed on interpreting the meaning of the height of each bar and the sum of the heights of all the bars. Students should work in groups to complete and discuss the results of the exploration.
- Review the properties of normal curves and the empirical or 68-95-99.7 rule related to how data is position in a normal distribution. Use the Normal Distribution Practice sheet to complete *Exercise #1.*
- *Exploration #2* focuses on finding area under a normal curve and interpreting the associated probability. The exercise can be done with the Standard Normal Probabilities Table or a calculator. This should like begin as a whole class activity since the analysis along with reading a z-table is new.
- *Exploration#3* Students should work in pairs to discuss strategies they might use to identify the mean and standard deviation for each graph.

Assessment

- **Questions**
  - What are some ways area under a normal curve is interpreted?
  - Why is the area under a normal curve equal to one?

- **Journal/writing prompts**
  - When we determine that a data set is "normally distributed", it is based upon some statistical testing that you don't know at this point, but without that, what kinds of things would you look for in a data set that might indicate it is normally distributed? Provide examples and rationale.
  - Describe a z-score in your own words.

Strategies for Differentiation

- Chunk information into smaller groups and teach the topic over a longer period of time.
- Create a human line plot according to students' heights and discuss how 'normal' it is.
- Students create flash cards with vocabulary / description or picture.
- Use an interactive whiteboard to demonstrate shading under normal curve and to model use of a Standard Normal Table.

Statistics Review

1. Given the data set: {13, 10, 2, 2, 4, 12, 8, 6, 5, 9, 11, 14, 11, 8, 5, 8}

a. Find the mean, median and mode.

b. Add two different data values to the set that won't affect the mean, median or mode.

c. Construct a histogram of the data including the data values you added.

d. Using a calculator, find the standard deviation of the data set including the new values.

e. Which values are within 1 standard deviation of the mean? Are any data values more than 2 standard deviations from the mean?

f. Add two more data values, one above and one below the mean, which will increase the standard deviation. Calculate the new standard deviation.

z-score review – A z-score indicates the location of a data value relative to the mean in terms of standard deviation units.

2. Given a data set has a mean, $\mu=10$, and a standard deviation, $\sigma=2$, a data value of 12 would have a z-score of 1 and a data value of 8 would have a z-score of -1 (why?).

a. What z-score would be assigned to 6? 14? 5? 20?

b. What data value would have a z-score of -3? 4? 0? 2.5?

c. Write a formula to determine the z-score for any value, X, in this data set.

An experiment consists of flipping 4 coins and recording the number of heads.

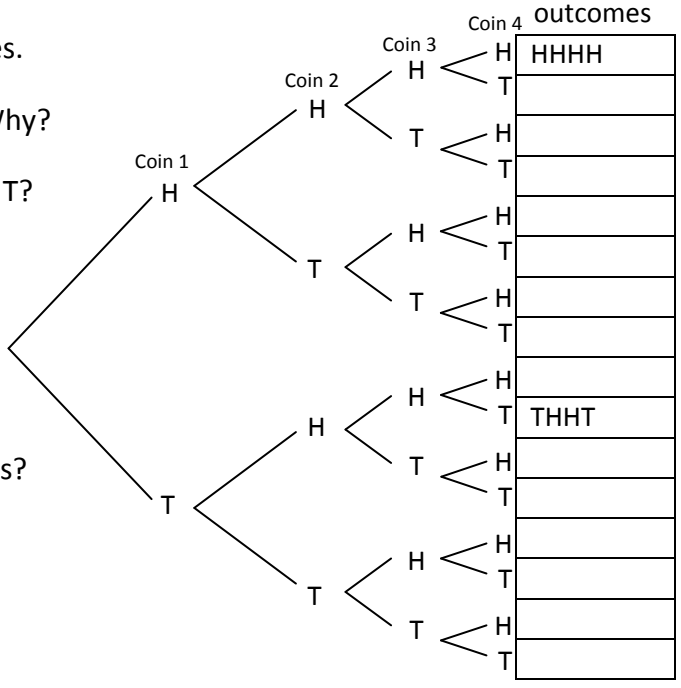
a. Complete the list of possible outcomes.

b. Are all outcomes are equally likely. Why?

c. What is the probability of getting THHT? TTHH? HHHH?

d. What is the probability of getting exactly 4 heads? 2 heads and 2 tails? 1 head and 3 tails?

e. What is the sum of all the probabilities?



Exploration #1 – What is Normal? What makes Normal Curves Different?

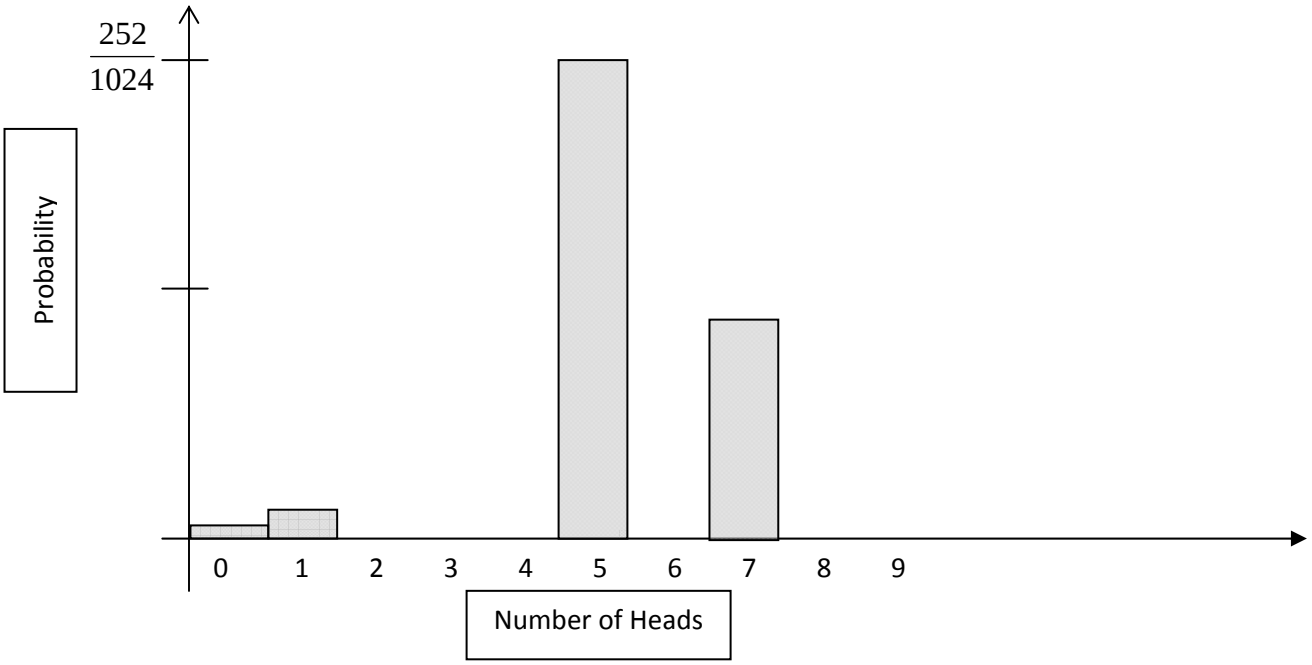
Experiment – Flip 10 coins 1024 times and count the number of heads. You can test this if you want, but let's first focus on the theoretical probabilities.

Using *combinations*, we can obtain the expected values and theoretical probabilities in the table below:

Number of Heads	Expected Frequency value out of 1024	Theoretical Probability	Percent Likelihood
0	1	$\frac{1}{1024}$	
1	10	$\frac{10}{1024}$	
2	45	$\frac{45}{1024}$	
3	120	$\frac{120}{1024}$	
4	210	$\frac{210}{1024}$	
5	252	$\frac{252}{1024}$	
6	210	$\frac{210}{1024}$	
7	120	$\frac{120}{1024}$	
8	45	$\frac{45}{1024}$	
9	10	$\frac{10}{1024}$	
10	1	$\frac{1}{1024}$	

Exploration #1

a. What is the sum of all the probabilities?_____
b. Complete the percent likelihood by converting the theoretical probability to a decimal.
c. What observations can you make about the data in the table so far?
d. On the axis provided complete the histogram of the theoretical probability for each number of heads.



e. Draw a point at the midpoint of the top of each bar.
f. Connect the data points with a smooth curve
g. What do you observe about the graph's:
   - Shape?
   - Symmetry?
   - Highest point?
   - Mean/median/mode probability?
h. Read about the characteristics of a Normal Curve and discuss how the curve you drew compares to those.

The Graph of a Normal Distribution is a Normal Curve, and every normal curve has the following characteristics:

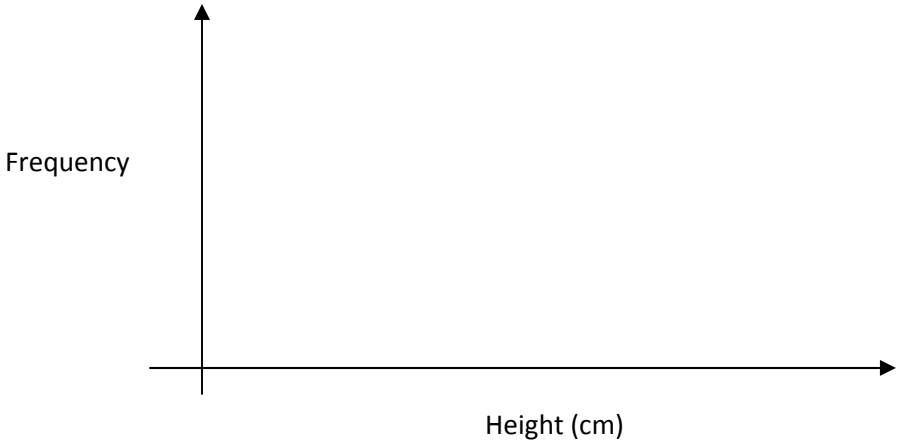
- The mean, median and mode are equal.
- They are bell-shaped and symmetrical about the mean.
- The curve never touches, but it closer and closer to the x-axis as it gets farther from the mean.
- The total area under the curve is equal to 1.

Exploration #2 –

The heights of all 4th grade students in a particular school and the frequency are in the table below:

Height (cm)	Frequency
130	2
131	7
132	9
133	20
134	38
135	18
136	12
137	10
138	5

a. Construct a histogram of the data using axis provided.



b. What percentage of the students are shorter than 135 cm?

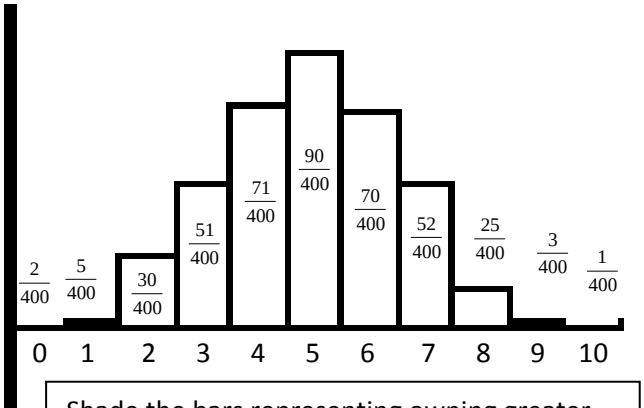
c. If one student was selected at random, what is the probability the height would be greater than 132 cm but less than 138?

d. How many 4th grade students are represented in the data?

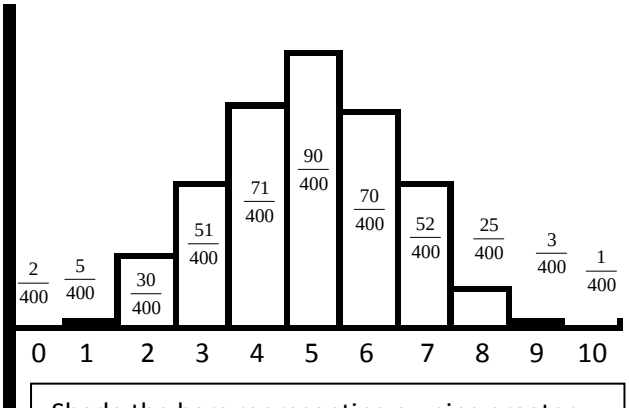
e. What is the mean height of the data set?

f. Does the data appear to be normally distributed? Why or why not?

Shading the probability – The graphs below reflects the number of pets veterinarians own. The value associated with each bar represents the fraction of veterinarians with that many pets.



Shade the bars representing owning greater than or equal to 5 pets. What fraction has 5 or more pets?



Shade the bars representing owning greater than 2 but less than 7 pets. What fraction of owners falls into this group?

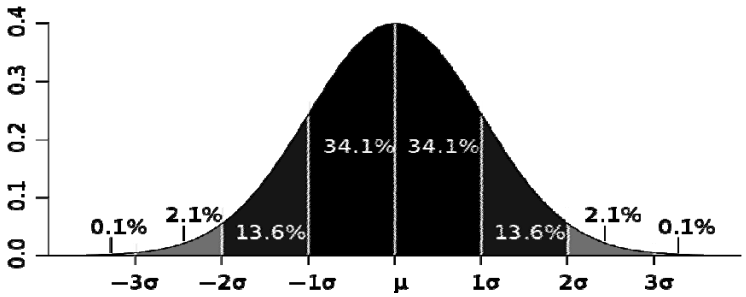
Exercise #1

Using the normal distribution graphs on + *Normal Distribution Practice* sheet, represent the following on one of the graphs. Show three standard deviations to the left and right of the mean.

1. A normal distribution with a mean of 7 and a standard deviation of 2.
2. A normal distribution with a mean of 500 and a standard deviation of 100.
3. The weights of cattle at the fair this year were normally distributed with a mean of 800lbs with a standard deviation of 65 lbs.
4. The amount of time a middle school student studies in a night is normally distributed with a mean of 30 minutes with a standard deviation of 7 minutes.
5. The length of hair of a private in the army is normally distributed with a mean of 1cm and a standard deviation of 0.3cm.
6. The length of wear on Spinning Tires is normally distributed with a mean of 60,000 miles and a standard deviation of 5,000 miles. Shade the region under the curve that represents the fraction of tires that last between 50,000 miles and 70,000 miles. What fraction of tires does that represent?
7. The number of crackers in a box of Crackerbox Crackers is normally distributed with a mean of 75 and a standard deviation of 2. Shade the region under the curve that represents the probability that a box has between 73 and 77 crackers. What is that probability?
8. The length of time it takes to groom a dog at Shaggy's Pet Shoppe is normally distributed with a mean of 45 minutes and standard deviation of 10 minutes. Shade the region under the curve that represents the percent of dog grooming times between and 65 minutes. What is that percent?

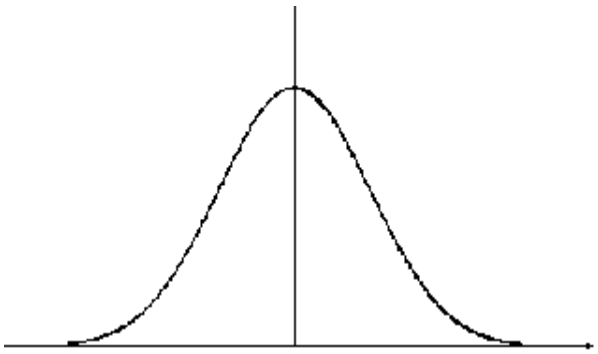
Exploration #2

Finding area under a normal curve



Areas can be found under a normal curve using the 68-95-99.9 Rule if the areas are bounded at places where an exact standard deviation occurs. Areas that aren't bounded at specific standard deviation units, can be found using a calculator or a z-table.

#1 A corn chip factory packs chips in bags with normally distributed weights with a mean of 12.4 oz with a standard deviation of 0.15 oz.



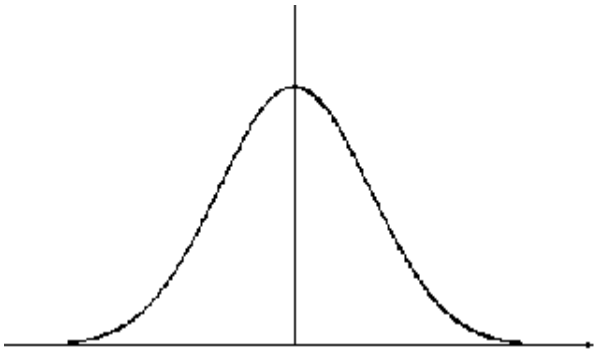
a. Label the mean and 3 standard deviations above and below the mean on the graph provided.

b. Shade the region that indicates the percentage of bags that contain less than 12.64 oz.

c. Determine the z-score corresponding to 12.64.

$$z-\text{score}=\frac{X-\mu}{\sigma}$$

d. Use the Standard Normal Probabilities Table to find the area associated with the z-score obtained in c and interpret your result.



e. Label and shade the region that represents the likelihood a bag will contain between 12.1 and 12.76 oz.

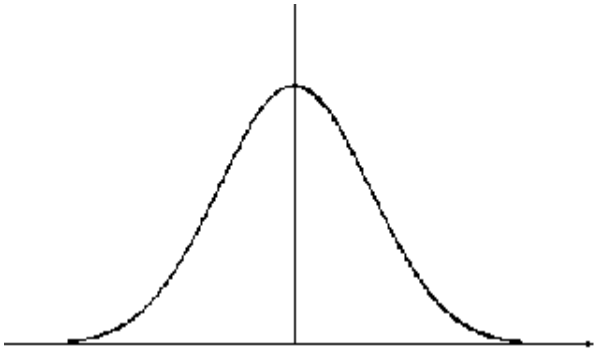
f. Calculate the z-scores corresponding to both 12.1 and 12.76 and find the Standard Normal Probabilities for each using a calculator the Standard Normal Probabilities Table.

g. Determine how you would use those values to determine the probability a bag chosen at random would contain between 12.1 and 12.76 chips.

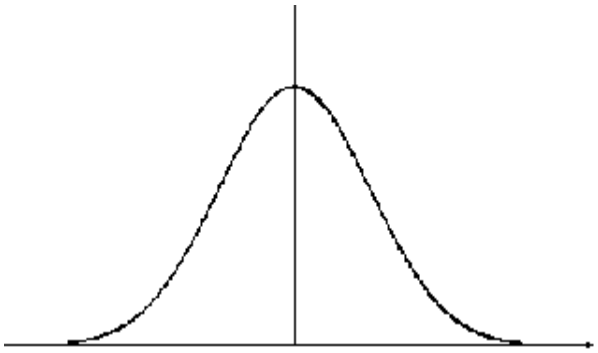
Exploration #2 cont.

#2 The length of adult unicorns' horns are normally distributed with a mean of 10.1 cm and a standard deviation of 1.04.

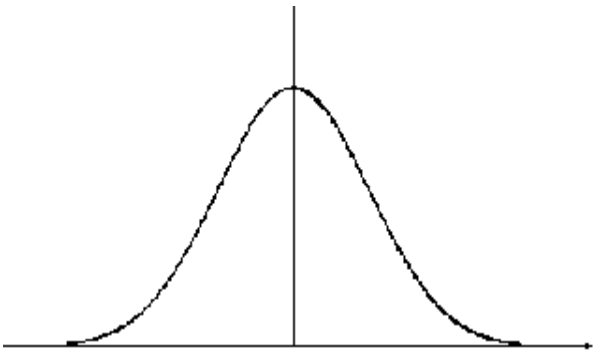
a. Label the mean and 3 standard deviations above and below the mean on the graph provided.



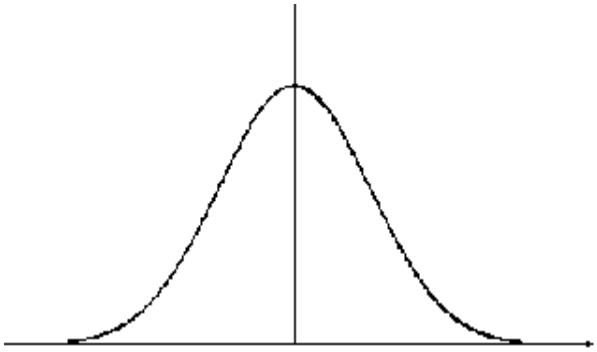
b. What percent of adult unicorns have horns shorter than 10.1 cm? longer than 10.1 cm? (exactly 10.1? You need to create this normal curve on your calculator and find the value of y when x=10.1. Your teacher should be able to help)



c. Shade the region on the adjacent graph that represents the probability a unicorn's horn is longer than 9 cm. Calculate the area of that region by finding the appropriate z-score and using the Standard Normal Probabilities Table or a calculator. INTERPRET YOUR RESULT.



d. Shade the region on the adjacent graph that represents the probability a unicorn's horn is longer than 10.5 cm or less than 9.5 cm. Calculate the area of the associated regions by finding the appropriate z-score and using the Standard Normal Probabilities Table or a calculator. INTERPRET YOUR RESULT.



e. Challenge: In order for a unicorn to be admitted to college, she horn must be in the 75^{th} percentile. That means 75% of the unicorn horns must have horns shorter than hers. Shade the region representing this area and determine the value associated with the 75^{th} percentile.

Exploration **#3**

Each of the following normal curves are graphed in the same viewing window on a graphing calculator. The mean is either 7, 8, 9, or 10. The standard deviation of each is 1, 1.5, 2, or 2.5

Use that information to determine the mean and standard deviation of each graph. Check you answers on the viewing window:

```
WINDOW
 Xmin=4
 Xmax=16
 Xscl=1
 Ymin=0
 Ymax=.4
 Yscl=1
 Xres=1
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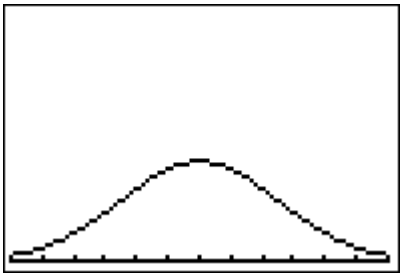
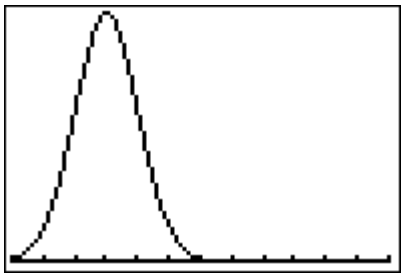
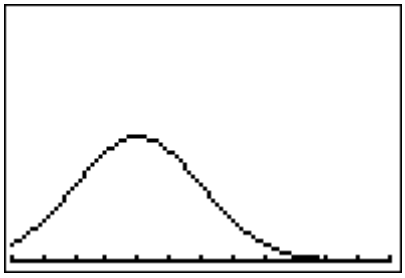


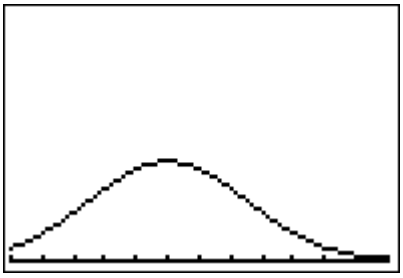
μ= σ=



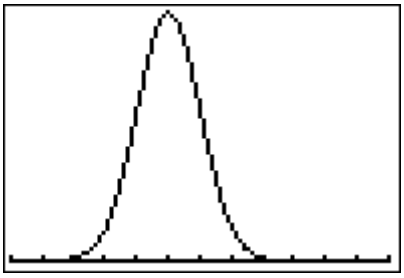
μ= σ=



μ= σ=



μ= σ=



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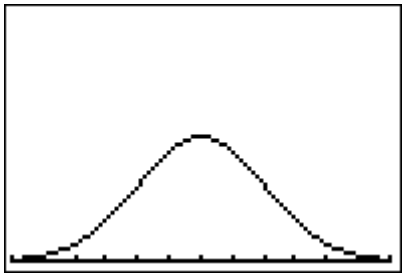
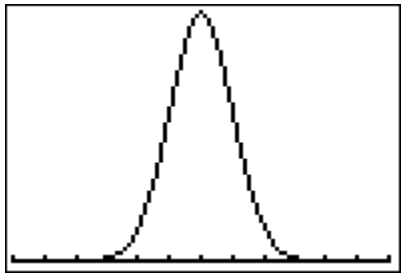
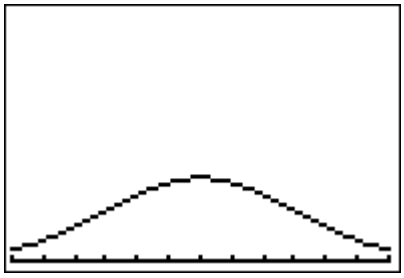


μ= σ=



μ= σ=



μ= σ=

Exercises

1. The College of Knowledge gives an admission qualifying exam. The results are normally distributed with a mean of 500 and a standard deviation of 100. The admissions department would only like to accept students who score in the 65^{th} percentile or better. Complete the chart below and then determine which students would qualify and what score is associated with the 65^{th} percentile. Which students qualify for admission?

Student Score	z-score	percentile
530		
570		
650		
800		
540		

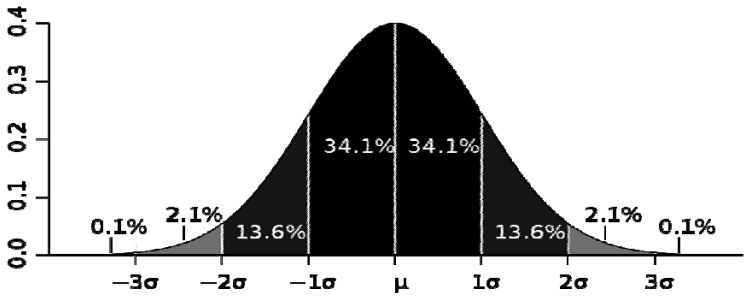
2. The MP3 Player made by Mango Corp., aPod, has an average battery of 400 hours. Battery life for the aPod is normally distributed with a standard deviation of 25 hours. The MP3 player made by Pineapple Inc., the PeaPod, has an average battery life of 390 hours. The distribution for its battery life is also normally distributed with a standard deviation of 30 hours.

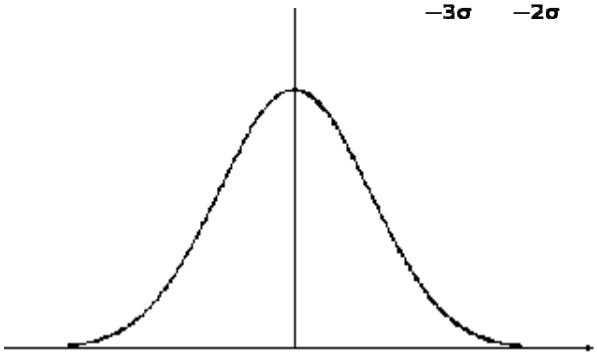
   a. Find the z- scores for each battery with lives of 250, 350, 410, and 450 hours.
   b. Which battery lasting 410 hours performed better?
   c. What percent of the aPod Batteries last between 375 and 410 hours?
   d. What percent of PeaPod batteries last **more** than 370 hours?

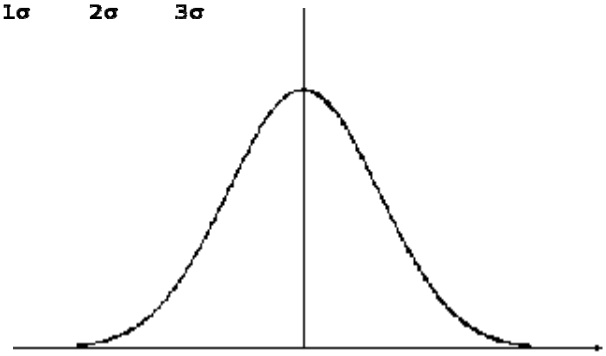
3. **No Calculator / No table:** The braking distance for a Krazy-car traveling at 50 mph is normally distributed with a mean of 50 ft and a standard deviation of 5 ft.

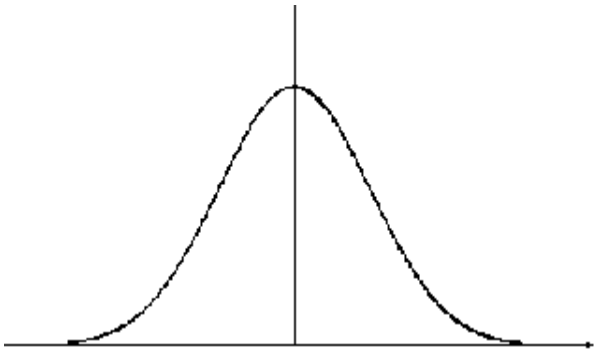
   a. What is the likelihood a car will take more than 65 feet to stop?
   b. What is the probability a car will stop between 45 ft and 55 ft?
   c. What percent of the time will a Krazy-car traveling at 50 mph stop between 35 and 55 ft.?
   d. What is the probability a car will require less than 50 or more than 60 ft. to stop?

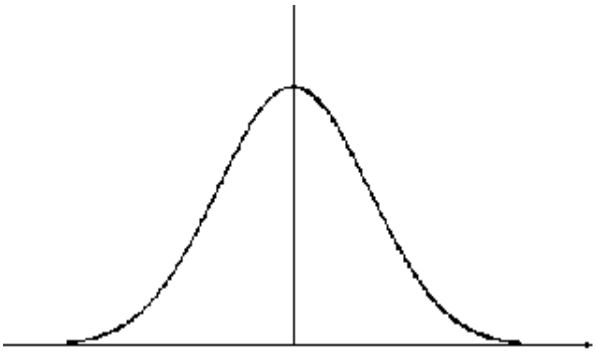
Normal Distribution Practice

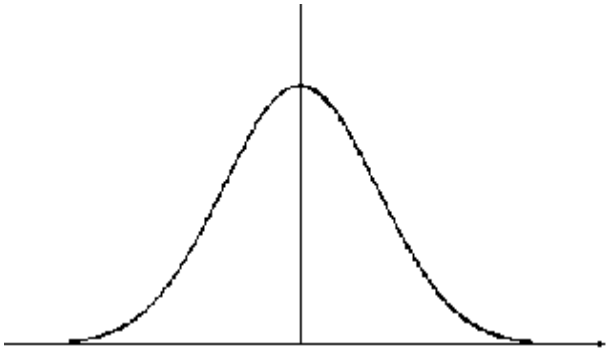


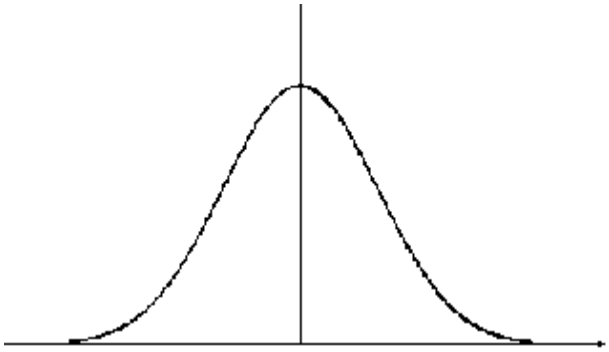


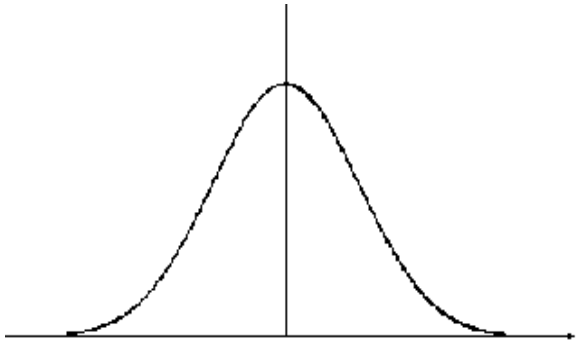


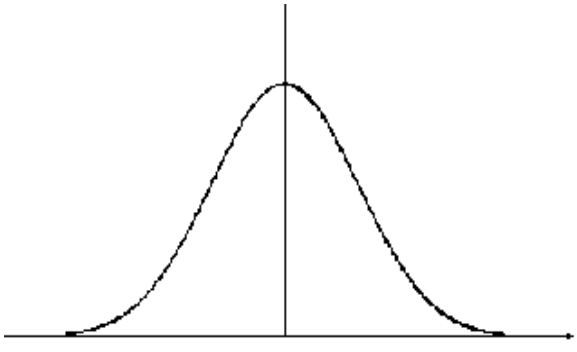












Standard Normal Probabilities

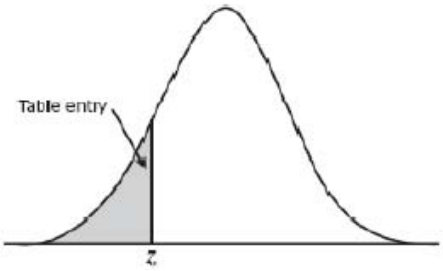


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

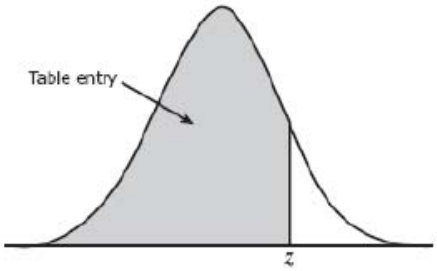


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998